Testing of Materials for Analysis of Fatigue and Elastic Properties

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In this experiment, three different metals underwent uniaxial tensile tests where we obtained the stress vs. strain properties of the materials. The three metals, aluminum, brass, and copper had a Young's modulus of 68.1, 85.1, and 26.1 GPa respectively. They also shad similarly ranked yield stresses, ultimate stresses, and fracture stresses. By numerically integrating each of their stress-strain curves, we obtained their moduli of resilience and moduli of toughness. The moduli of resilience for aluminum, brass, and copper were 340.5, 1905.9, and 183.1 MPa, respectively and their moduli of toughness came out to be 14.432, 23.317, and 4.539, respectively. A rotary fatigue test was done for an Aluminum specimen at different loads. The corresponding stresses were calculated using stress equations and plotted against the number of cycles each test withstood until fatigue, logarithmically to obtain the S-N curve for the material. This gave the fatigue properties of the material so that it can be estimated how many cycles it would take at certain cyclic stresses for the material to fracture.

INTRODUCTION

The material used in specific applications is crucial to its effectiveness and longevity of its use before fracturing. There are many factors that engineers must look at to ensure their design criteria are met, which include: thermal, magnetic, electrical, optical, acoustic, and mechanical properties, like tensile behavior, ductility, hardness. Along with all of this, cost, availability, and durability play a big factor in the practicality of the material in the use of it in mechanical systems.

Mechanical properties are revealed when subject to a force system. The temperature and loading rate play a big role on the material behavior. This behavior is further broken down into two categories: elastic and inelastic (plastic) deformation.

Most metallic systems (polycrystalline structures) are linear isotropic solids, which means they are elastic and are characterized by a Young's modulus, given by Equation 1. This value is the ratio of the stress to strain when an axial load is applied to the material like during a uniaxial test.

$$E = \frac{\sigma_{axial}}{\varepsilon_{axial}} \qquad [1]$$

$$\upsilon = -\frac{\varepsilon_{lateral}}{\varepsilon_{axial}} \qquad [2]$$

Another material property of elastic materials is a Poisson ratio, given by Equation 2, which relates the lateral strain to the axial strain during the uniaxial test. Most metals have a Poisson's ratio of around 0.3. The magnitudes of these ratios give a lot of useful information to engineers because it relates the materials' properties to the magnitude of the applied load that is occurring.

Along with elastic properties, materials also have inelastic properties that are measured using values such as yield stress, ultimate stress, fracture stress, elongation, toughness, and ductility. These properties of a material can easily be illustrated with a stress-strain curve shown in Figure 1.

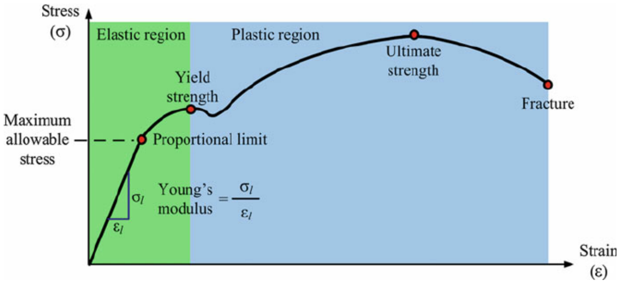

Figure 1. Engineering Stress-Strain curve

In the diagram, we have Young's modulus as the slope elastic region and the proportional limit as the point where the curve goes from linear (elastic) to nonlinear (plastic) deformation. Before the proportional limit, if the stress is relieved, the material is able to recover to its original length and relative shape.

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Between the proportional limit and the yield stress, the material is going through plastic and elastic deformation. If the yield stress cannot be seen easily from the stress-strain curve, a convention that can be used to find it is the 0.2% conventional yield stress. This method says that the yield stress intersects the line, which starts at 0.2% strain and has a linear slope of the Young's modulus. This line is parallel to the elastic portion and shifted to the right by 0.2% strain.

After the yield stress, the material deforms only plastically and it reaches the ultimate stress, which is the highest stress that the material withstands. After the ultimate stress, the material starts to neck, meaning the cross sectional area at the middle decreases, and ultimately cracks at the fracture stress, the last stress it has. In the whole process the whole material is getting longer, which is why the term elongation to fracture is suitable. It measures the relative elongation of the material at fracture with respect to the original length.

The material's modulus of resilience is defined as the area under the elastic (linear) region of the curve and represents the energy that the material can absorb without creating a permanent distortion. The material's modulus of toughness is the total energy that the material can absorb before fracturing, so it is equal to the area under the entire stress-strain curve.

In practice, a conventional and common way to obtain all this information is through a uniaxial tensile test, which is what we will be performing with three materials: copper, brass, and aluminum. In the test, the material of known cross sectional area and length is stretched at a predetermined rate and the load and elongation are recorded so the stress and strain during the procedure can be obtained.

Next, we look at the fatigue of a material using the rotary fatigue test. Fatigue the weakening of a material caused by repeatedly applied loads, even below its yield stress. Cyclic loading and unloading of a material causes microscopic cracks to form and eventual leads to the failure of the material. The fatigue strength or fatigue limit is the amplitude of cyclic stress that can be applied to a material without causing failure. Fatigue life is defined as the number of cycles that a given material can withstand a specific cyclic stress amount until failure. The fatigue strength for steels is usually around 0.5 times the ultimate tensile strength of the material and for aluminum, copper, and iron alloys it is around 0.4 times the ultimate tensile strength. An S-N curve can help illustrate these values as it shows the fatigue life, or number of cycles the material can withstand at a range of stresses and an example is shown in Figure 3.

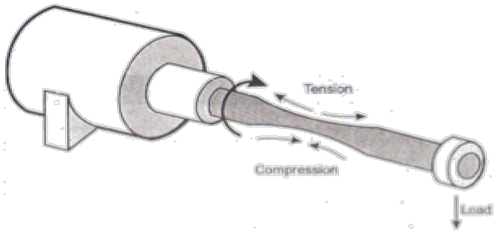

Figure 2. Rotary Fatigue Machine

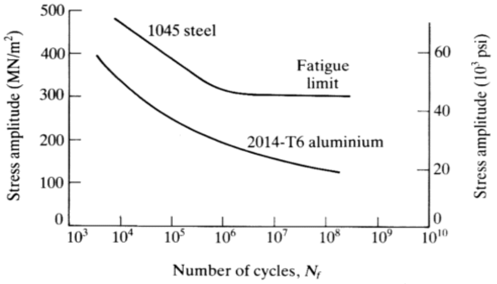

Figure 3. S-N Curve for Fatigue

The rotary fatigue machine consists of a cantilever beam that is bending due to an applied load at its end. The machine rotates the beam to cycle tension and compression on the beam. Equations 3 and 4 give the bending stress at a point and second moment of area for the circular cross-sectional beam, respectively, and the result of both equations combined is the standard bending equation given by Equation 5.

$$\sigma = \frac{My}{I_x} \quad [3]$$

$$I_x = \frac{\pi D^4}{64} \quad [4]$$

$$\sigma = \frac{FL \cdot 32}{\pi D^3} \quad [5]$$

RESULTS AND DISCUSSION

In the experiment, we first measured the dimensions of the three metal specimen using Vernier Calipers (dimensions tabulated in Appendix). Then, we use the clamp system with the PASCO software to measure and record the time, position, force, and speed during the experiment. Using this data, the stress and strain values were calculated, and we obtain the following stress-strain (S-S) curves for the three materials shown in Figures 4 to 6.

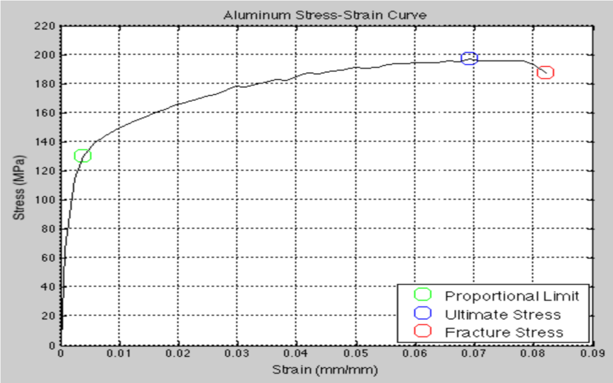

Figure 4. Brass Stress-Strain Curve

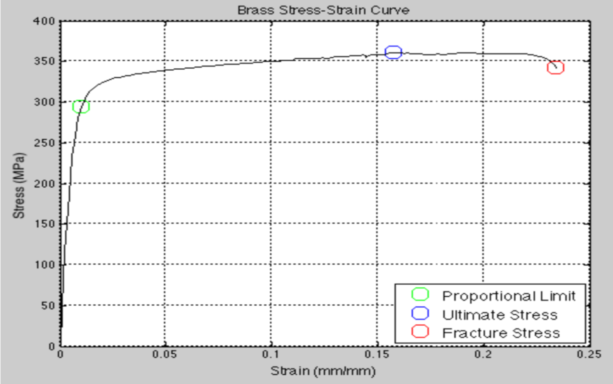

Figure 5. Brass Stress-Strain Curve

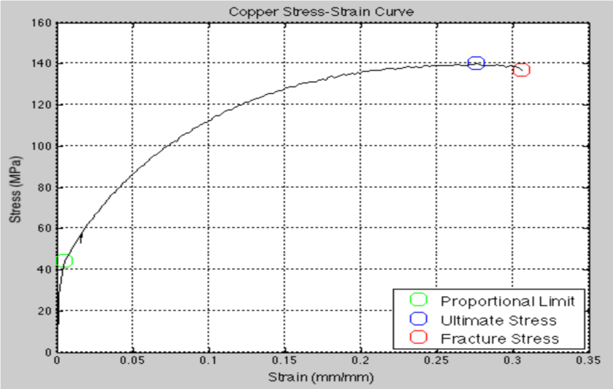

Figure 6. Copper Stress-Strain Curve

The proportional limit, ultimate tensile stress, and fracture stress are all labeled in Figures 4 through 6. The corresponding values are given in Table 1. We further analyze the elastic portion of the curve by zooming in to the three stress-strain curves. After seeing the linear portion of the graph as in Figures 7-9, we do a linear fit of the linear portion and get the slope of that linear fit; it is shown as the blue line. This corresponds to the Young's modulus of elasticity of the material. These values are also tabulated in Table 1. From these three graphs, we can approximate the yield stress of the material by using the 0.2% strain offset method mentioned in the introduction. As shown in Figures 7-9, the red lines next to the blue linear fit are of the same slope (Young's modulus), but start at 0.02 percent strain, rather than at 0. The intersection of this line and the stress-strain curve is the yield stress of the material. The yield stress is recorded in Table1.

Table 1. Material Elasticity & Strength Properties

Material	Young's Modulus (GPa)	0.2% Yield Stress (MPa)	Ultimate Stress (MPa)	Fracture Stress (MPa)
Aluminum	68.1	130.6	197.0	188.0
Brass	85.1	190.8	360.8	341.8
Copper	26.1	38.8	139.9	136.7

Figure 7. Linear Portion of Al S-S Curve

To much of our dismay, finding the linear region of these stress-strain curves was very difficult because for all the materials, the linear region only lasted around 1 to 2 data points, so the slope was not accurate at all. For aluminum, an approximation technique was used to interpolate another data point to get a better

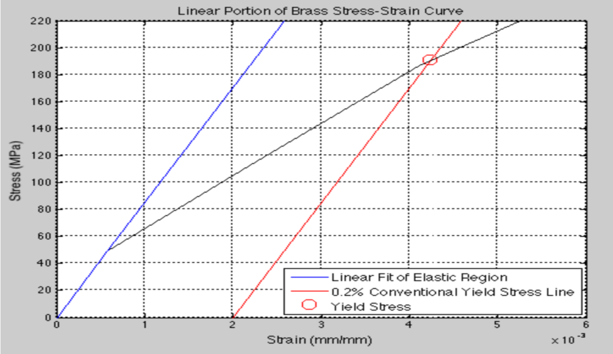

Figure 8. Linear Portion of Br S-S Curve

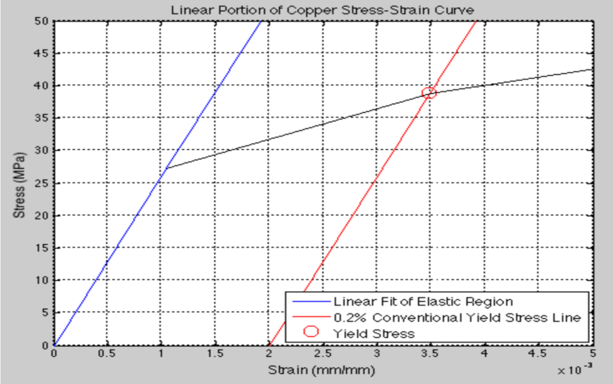

Figure 9. Linear Portion of Cu S-S Curve

estimate on the slope of the stress-strain curve. If the experiment were to be reiterated, the sampling frequency should be substantially increased, especially for the beginning, linear portion of the tensile test. Similarly, the load should be applied more uniformly without a human hand, but a motor that rotates the wheel at a slow, steady pace.

Next we use assess the modulus of resilience and toughness of the three materials by numerically approximating the area under the linear part of the curve and the entire S-S curve. The results are given in Table 2, while the code written to approximate the areas under the curves in MatLab are given in the Appendix. The general approximation involved a trapezoidal summation of all the trapezoids between each data point.

Table 2. Modulus of Resilience and Toughness

Material	Modulus of Resilience (kPa/m^3)	Modulus of Toughness (MPa/m^3)
Aluminum	340.5	14.432
Brass	1905.9	23.317
Copper	183.1	4.539

While the modulus of resilience can be found by simply using the proportional limit and getting the area of that region by treating it as a triangle, we decided this would not be accurate because the elastic region is only represented by 1-3 data points. Thus, we use the trapezoid rule for this too, so that the error is a little less. Still, the value does not match theoretical values too closely.

The modulus of resilience tells us how much energy the material can absorb without causing a permanent distortion to the material. As we can see, aluminum can absorb almost double the energy copper can, but they are both significantly lower than what brass can absorb. Next, the modulus of toughness represents the amount of work that must be applied per unit volume of a material to lead it to fracture under static loading. In this sense, similar to modulus of resilience ranking, brass requires the most work to lead it to fracture, followed by aluminum and then copper.

As discussed, the peculiar shape of the linear region of all three S-S curves is likely due to the low sampling frequency. Another peculiar behavior in these graphs is seen after the yield stress in the Copper S-S curve. Shortly after the yield stress is reached, the curve dips down a little and then goes back to increasing at a usual way. This dip may be due to human error, by a jerking motion while turning the wheel that applies the tensile load on the material. It can also be due to some other material property specific to copper, because it only happened for copper. In my opinion it is likely due to human error because the other materials did not show this behavior and there is not information about the stress and strain behavior than can readily explain this phenomena.

Next, we performed our fatigue test using Aluminum Alloy 2011 T6, whose dimensions are shown in the Appendix. We performed 4 tests under different loads and calculated the associated stresses using Equation 5 and the results are shown in Table 3. The corresponding S-N curve is shown in Figure 10.

Table 3. Fatigue Test Result at Cycle Rate 63Hz

Test #	Load (N)	Stress (MPa)	Cycle Count
1	77.8	346.70	48504
2	61.1	272.28	101197
3	50.4	224.60	401109
4	42.2	188.05	909887

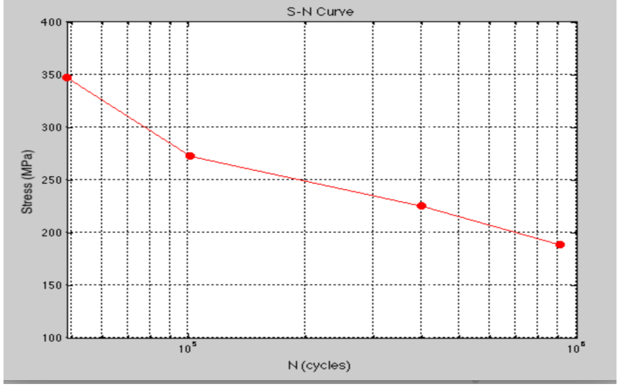

Figure 10. S-N Curve for Aluminum

In the S-N Curve, which shows the stress vs. cycle count until fracture of the material, the x-axis, which is the cycle count, is shown on a logarithmic scale. This is done to make it visually clear that when the slope of the curve tends to zero, that means the fatigue limit is reached, because as stress is decreased slightly, the cycle count it can withstand increases drastically and thus is reaching the fatigue limit. In the case of our experiment, the slope of the S-N curve does not tend to zero, or does not end off with a flat, horizontal curve. Thus, we can say that aluminum does not reach a fatigue limit. If more tests are done below 180, we may be able to get to the fatigue limit, which is known to be 124 MPa, according the MatWeb resource.

Based on this material data, we can conclude that certain materials are better suited for different functions. For example, copper has the lowest modulus of resistance and toughness. This shows that copper is very malleable because it does not absorb much applied energy to it and deforms easily. Furthermore, because brass absorbs the most energy without deforming, it is good for machine that undergo a lot of vibrations, so it can sustain the energy impact without deforming permanently. Aluminum has a relatively medium yield, ultimate, and fracture stress so it can be used for a combination of things.

CONCLUSIONS

The results of this lab show the way in which each material reacts to an increase in tensile loading. The elastic and inelastic properties are measured and quantified by analyzing their stress-strain curves. We see that as the ultimate stress is reached, the material starts to neck and the stress strain curve dips down until fracture. This shows that the atoms start to separate substantially in the middle and all the stress is at the point because of this stress concentration due to the necking. We get information like the Young's modulus, yield stress using the 0.2% conventional yield stress method, ultimate stress, and fracture stress. By numerically integrating the stress-strain curve, we obtain the modulus of resilience, and modulus of toughness for all the materials to great accuracy. If the tests were to be reiterated, some changes would easily make the results more accurate. These include applying a tensile load more slowly so that more data points can be taken at each stress/strain level for a more continuous curve. Also, if human interaction were cut out, the jerking motion of the crank would be eliminated for a steady load increase that a computer could easily maintain.

The second part of the lab consisted of a fatigue test and the results from this came out to be close to what we expected. The associated S-N curve decreases in slope, which shows that the material is reaching the fatigue limit. If we had more time and resources, we would do even more loads and lower stresses to see the curve level out and obtain the true endurance limit.

REFERENCES

[1] Hibbler, R. C. (2013). *Mechanics of Materials* (9th ed.). Boston: Pearson.

[2] Figliola, Richard S., *Theory and Design of Mechanical Measurements*, 5th ed, John Wiley and Sons, Danvers MA, 2011.

[3] Drazer, German, *Materials Testing Lab Manual*, Rutgers University, New Brunswick, 2017

APPENDIX

(1) Tensile Test Material Dimensions

Table 4. Tensile Test Material Dimensions

Material	Height (in)	Width (in)	Thickness (in)
Aluminum	2.407	0.2369	0.0281
Brass	2.471	0.2355	0.0315
Copper	2.398	0.2424	0.0381

(2) Method for Numerical Integration of Stress-Strain Curve for Modulus of Resilience and Modulus of Toughness

To get the modulus of resilience and modulus of toughness, we needed a way to calculate the area under the stress-strain curve to best accuracy possible. Because each stress data point from the computer is at a unique increment of strain, I decided traditional MatLab commands would not work, because they used set increments. I used the trapezoidal rule instead, where I would take the area of each trapezoidal section that the data points create and add the sum. This is shown with an example graph in Figure 11. For the modulus of resilience, this sum is taken only for the linear portion, up to the proportional limit. For the modulus of toughness, the area is taken in this way across the entire curve.

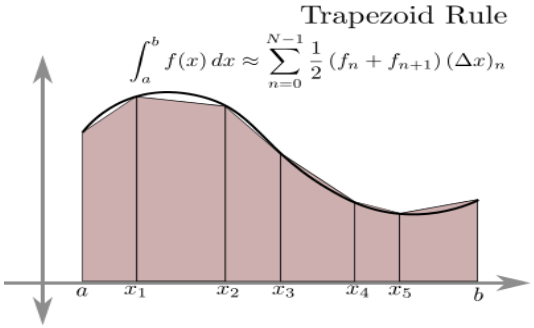

Figure 11. Trapezoid Rule

The corresponding code used to calculate the modulus of resilience, Amodr, and modulus of toughness, Amodt, are given below for aluminum. The same calculations are done for brass and copper.

<u>Code</u>:

```
Amodr = 0;
Amodt = 0;

for i = 1:4-1
        Amodr = Amodr +
        ((Astress(i+1)+Astress(i))/2)*(Astrain(i+1)-
        Astrain(i));
end

for i = 1:length(Astress)-1
        Amodt = Amodt +
        ((Astress(i+1)+Astress(i))/2)*(Astrain(i+1)-
        Astrain(i));
end
```

(3) Fatigue Test of Aluminum Properties & Dimensions

Tensile Strength: 395 MPa
Min. Yield Stress: 220 MPa
Theoretical Fatigue Limit: 124 MPa
L= 28 mm
D = 4 mm