Momentum Deficit Behind a Cylinder under Water Modeled in a Wind Tunnel

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A water flow across a long cylindrical rod of width 0.5" is modeled with an air flow in a wind tunnel with an equivalent Reynolds number to estimate the drag force on the water flow. The velocity of the wind tunnel is set to have the same Reynolds number as the water system. A velocity profile for air in a wind tunnel is obtained for the free-stream flow, flow 5.5" behind the cylinder, and 13.56" behind the cylinder using a hot wire anemometer that uses convection cooling of an electrically heated wire with a resistance being a function of the cooling rate, which is related to the wind speed. The velocity profiles are obtained through a conversion of the voltage profiles using the hot wire anemometer calibration curve. These profiles are used to obtain an average drag force of 0.1792 N/m and that is used to get an average drag coefficient of 0.7752, which are calculated by doing a momentum and mass conservation on the control volume of the model to get drag force and then non-dimensionalizing the drag force to get the drag coefficient. Using the fact that the model and system drag coefficient are equal because of the same Reynolds number and geometry used, we convert this model drag force to the system drag force and conclude it to be 0.5998 N/m.

INTRODUCTION

Some fluid motions are harder to analyze than others due to limitations in location, measurement devices, scale, and many other reasons. The solution that fluid scientists came up with is the use of the non-dimensional Reynolds number, which governs the motion of the underlying fluid conditions. If the Reynolds numbers for two fluid flows are the same, then both flows share similar characteristics

In this lab, the system which we wish to assess is an underwater installation including a long 0.5" OD rod which is subjected to a drag force from a 2.0 mile/hour flow of 40 °F water normal to its axis, but getting the drag force of this system is difficult because recreating the exact conditions and being able to measure this velocity is difficult. This is why we model this flow by obtaining the Reynolds number of this system flow using Equation 1 and recreating a flow of air in a wind tunnel with a different velocity to get the same Reynolds number. In Equation 1, Re is the Reynolds number, rho is the fluid density, V is the velocity, L is the length of the rod, mu is the dynamic viscosity, and nu is the kinematic viscosity. [1]

$$Re = \frac{\rho V L}{\mu} = \frac{VL}{\nu} \qquad [1]$$

The wind tunnel used in the experiment is shown in Figure 1 and it consists of a static pressure tap and manometer that is used to set the uniform incoming velocity of the airflow. It also includes a Dantec hotwire anemometry system to measure local flow velocities. The wind speed is related to the convective cooling on the electrically heated conducting wire, which is related to the resistance of the wire. This is used in the lab to measure the velocities of the flow at different heights behind the rod to get an approximate velocity profile behind the rod as a result of the drag force on the rod.

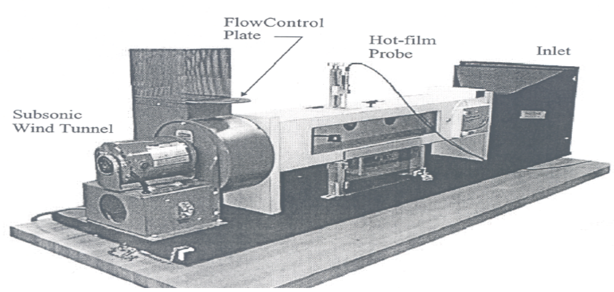

Figure 1. Wind Tunnel. [3]

The resulting velocity profile behind the rod is shown in Figure 2 and can be used to obtain the drag force induced by the rod.

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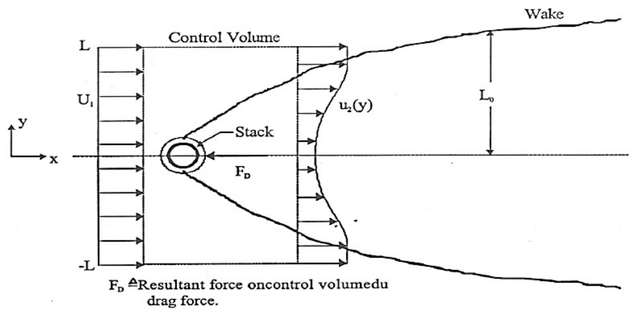

Figure 2. Schematic diagram showing the control volume and the wake behind the cylinder. [3]

By doing a conservation of mass and momentum in the wind tunnel control volume shown in Figure 2, we can derive Equation 2 (Derivation shown in Appendix) [1,3]

$$F_D = \rho w \int_{-L}^{L} u_3(y)[U_1 - u_3(y)]dy \quad [2]$$

The corresponding drag coefficient can then be derived into Equation 3.

$$C_D = \frac{F_D}{\frac{1}{2}\rho A V^2} \quad [3]$$

After obtaining the drag coefficient, we can set it equal to the drag coefficient for the system, because they are equal at the same Reynolds Number and obtain the relation between the model drag force and the system drag force given by Equation 4.[3] The propagation of error from the velocity measurement to the drag force is given by Equation 5.[2]

$$F_{System} = \rho_S A_S (U_S)^2 \frac{F_M}{\rho_M A_M (U_M)^2} \quad [4]$$

$$u_R = \left[\sum_{i=1}^{L} (\theta_i u_{\bar{x}_i})^2\right]^{1/2} \quad [5]$$

RESULTS AND DISCUSSION

Using Equation 1, we obtain a Reynolds Number of $1.2687557 * 10^4$ for the system and the corresponding velocity needed in the model to match this is 45.8 ft/s or 13.96 m/s. We calibrate the hot wire anemometer at different wind tunnel speeds using a voltmeter connected to the hot wire anemometer and use a spline fit to extrapolate the calibration curve shown in Figure 3. The calibration curve is not linear, but seems to be slightly exponential.

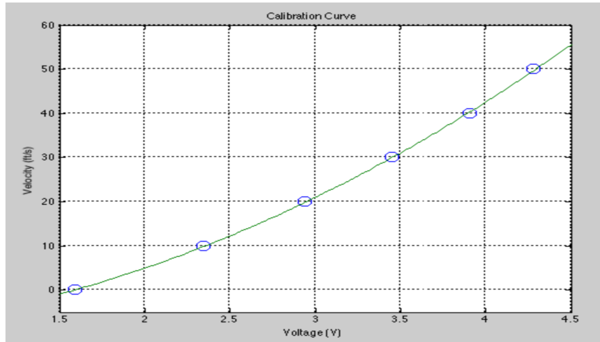

Figure 3. Calibration Curve.

Using this calibration curve, we find the respective velocities for the measured voltage on the horizontal location interval [-L,L]. The velocity profiles for the free-stream flow, flow measured 5.5” behind the rod, flow measured 13.56” behind the rod, and the average of these two locations is shown in Figure 4. As you can see, the free stream velocity is relatively constant throughout the height of the wind tunnel cross section. The velocity profile at 5.5” behind the cylinder is more reduced by the cylinder drag force that at 13.56” and the average lies between the profiles at these two stream-wise locations.

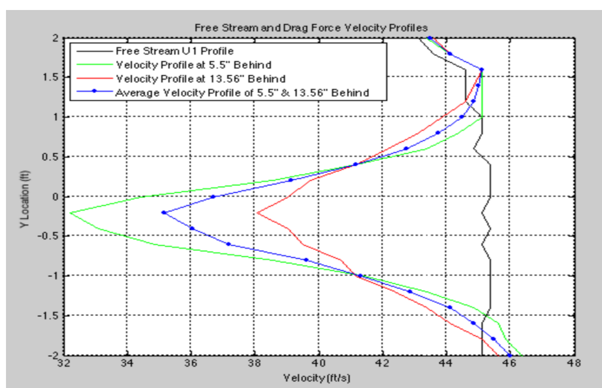

Figure 4. Velocity Profile of Free-Stream Flow, Probe placed 5.5” behind Cylinder, Probe placed 13.56” behind Cylinder, and Average of 5.5” and 13.56” behind Cylinder.

Table 1. Drag Force and Coefficient

Stream Wise Location (in)	Drag Force N	Drag Coefficient
5.5	0.2010	0.8694
13.56	0.1574	0.6810
Average	0.1792	0.7752

The drag force and coefficients are then calculated at both stream-wise location and are shown in Table 1 along with the average of each.

The calculated drag coefficient from Table 1 is around the same as the corresponding drag coefficient in Figure 5 with Reynolds number around 12,587.[1]

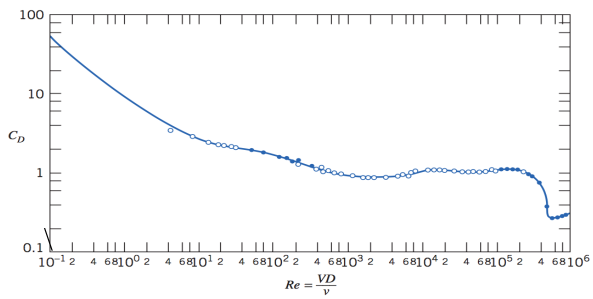

Figure 5. Drag coefficient for a smooth solid cylinder as a function of Reynolds number.

Although the drag coefficient is close, it is a little lower than the experimental results in the graph and this can be due to an error in voltage measurement that made velocity bigger it is and/or from errors in the approximation of the integral in the drag force equation.

Finally, using Equation 4, we obtain the drag force of the system we are observing: 0.5998 N/m.

In our analysis of the error associated with the experiment, we start with having a set 1% uncertainty in the velocity measurements, so the free-stream velocity is $13.7 \pm 0.137\ m/s$. Equation 5 gives the error propagation to the force and this is used to obtain a drag model force of $0.1792 \pm\ 0.0067 N/m$, which is a 3.74% error. This is a moderately significant uncertainty margin, which makes sense because the drag force is highly dependent on the velocity of the flow. This is further propagated to get the uncertainty of the drag force of rod in the water. Using Equation 5 again with respect to the free stream model velocity and the model drag force, we obtain a system drag force of $0.5998 \pm 0.0254\ N/m$. This is a reasonable amount of uncertainty as it is around 4.2% of the value itself, yet it is a pretty good estimate considering a completely different fluid and setting was used to make that estimate.

CONCLUSIONS

A wind tunnel and hot wire anemometry system is used to model the flow of water across a cylindrical rod by setting flow conditions to replicate the same Reynolds number and by getting a velocity profile through localized voltage measurements with the hot wire anemometer and a voltmeter. Calculated values for drag force and drag coefficient obtained through equations using these obtained velocity profiles are close to accepted values in similar experiments and theoretical data, but approximations and instrumentation errors contributed slightly to vary results. Generally, this gave a good estimate of the system's drag force as the final value is close to theoretical values of similar flows.

REFERENCES

[1] Pritchard , Phillip J., *Introduction to Fluid Mechanics,* 9th ed, John Wiley and Sons, Danvers MA, 2015.

[2] Figliola, Richard S., *Theory and Design of Mechanical Measurements*, 5th ed, John Wiley and Sons, Danvers MA, 2011.

[3] Drazer, German, *Momentum Deficit Lab Manual*, Rutgers University, New Brunswick, 2017

APPENDIX

(1) Derivation of Drag Force, F_D:

Assumptions:

(a) Incompressible
(b) Steady State
(c) Pressure Forces at Surfaces 1 & 2 are equal and opposite
(d) Pressure Forces at Surfaces 3 & 4 are equal and opposite

Using the general momentum equation below and following the above assumptions gives:

(Sum of forces on the C.V.)
=
(net rate of momentum out of C.V.)
+
(rate of momentum change within the C.V.)

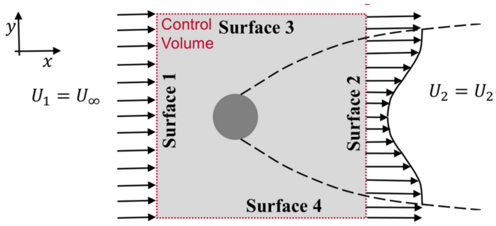

Figure 7. Control Volume with Surface Labels

Adding the forces and fluxes on all surfaces gives:
Force Equation:

$$-F_D = \text{w}\rho \int_{-L}^{L} U_1 U_1 \, dy + \text{w}\rho \int_{-L}^{L} u_2(y) u_2(y) \, dy + (\dot{m}_3 + \dot{m}_4) U_1$$

Mass Equation:

$$0 = \text{w}\rho \int_{-L}^{L} U_1 dy + \text{w}\rho \int_{-L}^{L} u_2(y) dy + (\dot{m}_3 + \dot{m}_4)$$

Multiplying the Mass Equation by $-U_1$ gives:

$$0 = -\text{w}\rho \int_{-L}^{L} U_1 U_1 dy - \text{w}\rho \int_{-L}^{L} U_1 u_2(y) dy - (\dot{m}_3 + \dot{m}_4) U_1$$

Adding this to the Force Equation gives,

$$F_D = \text{w}\rho \int_{-L}^{L} u_2(y)(U_1 - u_2(y)) dy$$

which agrees with the Equation 1 in the Lab Manual.[3]

(2) Numerical Integration of Drag Force, F_D, using Trapezoidal Approximation:

$$F_D \cong \text{w}\rho * \sum_{i=1}^{n} \{ dy * [u_3(i)(U_1 - u_3(i))) + u_3(i+1)(U_1 - u_3(i+1))] / 2 \}$$

where
n = length(u_3(y)) -1
dy = L(in inches) /n*0.0254 m = 0.00381 m
w = width of cylinder = 0.5*0.0254 m

This calculation is performed in MatLab using a for loop. The calculation for the Drag Force (F1) using the velocity profile of the probe 5.5" behind the cylinder in shown below. Similar code is used to numerically approximate the drag force at 13.56" behind the cylinder and the two results are averaged to get the true drag force, Fd.

```
dy = L/(size(U,2)-1);
Fd1 = 0;
for i = 1:(size(U,2)-1)
   s = ( (U(2,i)*(U1-U(2,i))) + (U(2,i+1)*(U1-
   U(2,i+1))) )/2;
   Fd1 = Fd1 + s * dy;
end
Fd1 = Fd1 * rhoAir * w;          % Fd1= 0.01385

-''-       % Similar code for Fd2 = 0.01231

Fd = (F1+F2)/2;           % Fd = 0.01308
```

(3) Example Calculations of Drag Coefficient, C_D:

$$C_D = \frac{F_D}{\frac{1}{2}\rho \, AV^2}$$

At 5.5" behind cylinder:

$$C_{D5.5} = \frac{0.2010}{\frac{1}{2}(1.2725)\,(0.5 * 6 * 0.0254^2)(13.7)^2}$$

$$C_{D5.5} = 0.8694$$

At 13.56" behind cylinder:

$$C_{D13.56} = \frac{0.1574}{\frac{1}{2}(1.2725)\,(0.5 * 6 * 0.0254^2)(13.7)^2}$$

$$C_{D13.56} = 0.6810$$

Average of values at both stream-wise locations gives

$$C_D = 0.7752$$